

Exercise 9

Differentiate the function.

$$g(x) = \ln(xe^{-2x})$$

Solution

Take the derivative of the function using the chain rule.

$$\begin{aligned} g'(x) &= \frac{d}{dx} [\ln(xe^{-2x})] \\ &= \frac{1}{xe^{-2x}} \cdot \frac{d}{dx} (xe^{-2x}) \\ &= \frac{1}{xe^{-2x}} \cdot \left\{ \left[\frac{d}{dx} (x) \right] e^{-2x} + x \left[\frac{d}{dx} (e^{-2x}) \right] \right\} \\ &= \frac{1}{xe^{-2x}} \cdot \left\{ (1)e^{-2x} + x \left[(e^{-2x}) \cdot \frac{d}{dx} (-2x) \right] \right\} \\ &= \frac{1}{xe^{-2x}} \cdot \{ (1)e^{-2x} + x [(e^{-2x}) \cdot (-2)] \} \\ &= \frac{1}{xe^{-2x}} \cdot (e^{-2x} - 2xe^{-2x}) \\ &= \frac{1 - 2x}{x} \\ &= \frac{1}{x} - 2 \end{aligned}$$